Source term identification in atmospheric modelling via sparse optimization

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Inverse modelling

- From observations $y \in \mathbb{R}^m$ we try to estimate release $x \in \mathbb{R}^n$
- We have (linear) model $M \in \mathbb{R}^{m \times n}$, which is known as sensitivity matrix
- Applications
  - Nuclear accidents (Fukushima)
  - Volcano eruption
  - Enforcing pollution limits

Solution approches

- Minimize the distance between y and Mx
- Add nonnegativity constraint on x
- Problem

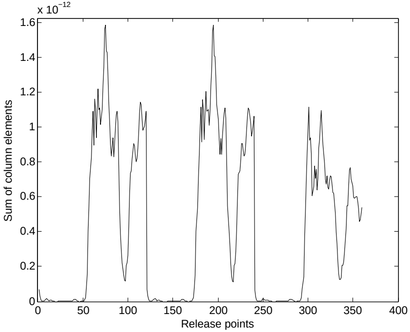
$$\begin{aligned} \text{minimize} \quad & \|Mx - y\|_2^2 \\ \text{subject to} \quad & x \geq 0. \end{aligned}$$

- Problem unstable. Some regulatization (Tikhonov) is usually added

$$\begin{aligned} \text{minimize} \quad & \|Mx - y\|_2^2 + \alpha \|x\|^2 \\ \text{subject to} \quad & x \geq 0. \end{aligned}$$

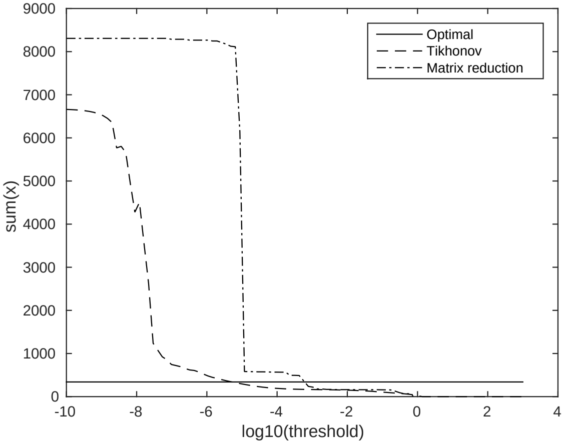
- Problematic interpretation?
  - Minimizing $\|Mx - y\|$ and $\|x\|$. Why the second part?
  - How to choose α?

- Denoting m_i columns of M, we have $Mx = \sum_{i=1}^{n} x_i m_i$
- Small norm of m_i may cause x_i to be huge
- Typical column norms of M



- Idea: omit columns with small norm

- Comparison of $\sum_{i=1}^{n} x_i$ for using Tikhonov regularization and removing some columns

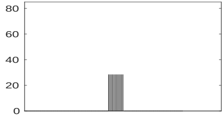


Sparsity

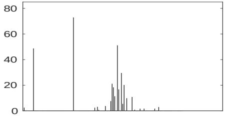
- Idea: look for a solution with high number of nonzeros
- Good interpretation for both identifying release point and release window
- Define $\|x\|_0$ to be the number of nonzeros and solve

$$\begin{aligned} \text{minimize} \quad & \|Mx - y\|_2^2 \\ \text{subject to} \quad & \|x\|_0 \leq k_{tol}, \\ & x \geq 0. \end{aligned}$$

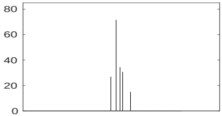
- Parameter $k_{tol} \in \mathbb{N}$ makes more sense than $\alpha > 0$



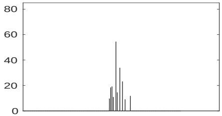
(a) Original solution



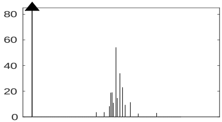
(b) OLS solution



(c) Sparse solution with $k_{tol} = 5$



(d) Sparse solution with $k_{tol} = 10$



(e) Sparse solution with $k_{tol} = 15$

Conclusion

- We have proposed to use the techniques of sparse optimization to the field of atmospheric modelling
- We modified the known methods to handle nonnegativity constraints
- The methods seem to have a good performace
- Submitted version available at Optimization Online
  `www.optimization-online.org/DB_HTML/2015/04/4861.html`