



## Source term identification in atmospheric modelling via sparse optimization

Lukas Adam (1), Martin Branda (1), and Thomas Hamburger (2)

(1) The Institute of Information Theory and Automation, Czech Academy of Sciences, Prague, Czech Republic  
(adam@utia.cas.cz), (2) Norwegian Institute for Air Research, Oslo, Norway (Thomas.Hamburger@nilu.no)

Inverse modelling plays an important role in identifying the amount of harmful substances released into atmosphere during major incidents such as power plant accidents or volcano eruptions. Another possible application of inverse modelling lies in the monitoring the CO<sub>2</sub> emission limits where only observations at certain places are available and the task is to estimate the total releases at given locations. This gives rise to minimizing the discrepancy between the observations and the model predictions.

There are two standard ways of solving such problems. In the first one, this discrepancy is regularized by adding additional terms. Such terms may include Tikhonov regularization, distance from a priori information or a smoothing term. The resulting, usually quadratic, problem is then solved via standard optimization solvers. The second approach assumes that the error term has a (normal) distribution and makes use of Bayesian modelling to identify the source term.

Instead of following the above-mentioned approaches, we utilize techniques from the field of compressive sensing. Such techniques look for a sparsest solution (solution with the smallest number of nonzeros) of a linear system, where a maximal allowed error term may be added to this system. Even though this field is a developed one with many possible solution techniques, most of them do not consider even the simplest constraints which are naturally present in atmospheric modelling. One of such examples is the nonnegativity of release amounts. We believe that the concept of a sparse solution is natural in both problems of identification of the source location and of the time process of the source release. In the first case, it is usually assumed that there are only few release points and the task is to find them. In the second case, the time window is usually much longer than the duration of the actual release. In both cases, the optimal solution should contain a large amount of zeros, giving rise to the concept of sparsity.

In the paper, we summarize several optimization techniques which are used for finding sparse solutions and propose their modifications to handle selected constraints such as nonnegativity constraints and simple linear constraints, for example the minimal or maximal amount of total release. These techniques range from successive convex approximations to solution of one nonconvex problem. On simple examples, we explain these techniques and compare them from the point of implementation simplicity, approximation capability and convergence properties.

Finally, these methods will be applied on the European Tracer Experiment (ETEX) data and the results will be compared with the current state of arts techniques such as regularized least squares or Bayesian approach. The obtained results show the surprisingly good results of these techniques.

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